

COMMON PRE - BOARD EXAMINATION - 2023

MATHEMATICS (041)

ANSWER KEY

CLASS: XII

MAX.MARKS: 80

SECTION A

Choose the correct answer:

1	If $x + \sin y = \log x$, then $\frac{dy}{dx} = \text{_____}$				1
	a) $\frac{\log x - 1}{\cos y}$	b) $\frac{x}{1 + \cos y}$	c) $\frac{x-1}{x \cos y}$	d) $\frac{1-x}{x \cos y}$	
2	The value of $x + y$ for which the matrix $A = \begin{bmatrix} 0 & -3 & 1 \\ 3 & 0 & -5 \\ x & y & 0 \end{bmatrix}$ is skew symmetric, is _____. 1				
	a) 4	b) -4	c) -6	d) 6	
3	If A is a square matrix of order 3 and $ A = 12$, then the value of $ A \ adj A $ is _____. 1				
	a) 12	b) 144	c) 1728	d) 27	
4	The positions of a kite at two different timings were noted and the equation of the line joining these two points was given as $x = -3, \frac{2y+4}{6} = 4z - 12$. The direction ratios of the line are: 1				
	a) (1, 6, 1)	b) (1, 3, 4)	c) (0, 6, 1)	d) (0, 3, $\frac{1}{4}$)	
5	If $f(x) = \log \sqrt{\tan x}$, then the value of $f^{-1}(x)$ at $x = \frac{\pi}{4}$ is _____. 1				
	a) 1	b) 0	c) ∞	d) $\frac{1}{2}$	
6	If A is a square matrix such that $A^2 = I$, then find the value of $(A - I)^3 + (A + I)^3 - 7A$ is _____. 1				
	a) A	b) I - A	c) I + A	d) 3A	
7	The value of λ when the projection of $\vec{a} = \lambda \hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units is _____. 1				
	a) 5	b) 7	c) ± 5	d) ± 7	
8	The corner points of the feasible region determined by a system of linear inequalities with $Z = 3x + 9y$ as objective function are A(0,20), B(15,15), C(5,5) and D(0,10). The maximum of Z: 1				
	a) occurs at only A	b) occurs at only B	c) occurs at A and B	d) occurs at every point on A	

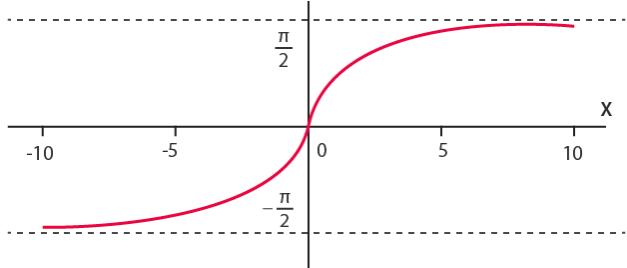
9	If $ \vec{a} = 8$, $ \vec{b} = 3$, $ \vec{a} \cdot \vec{b} = 12\sqrt{3}$, then find the value of $ \vec{a} \times \vec{b} $	1
	a) $4\sqrt{3}$ b) $12\sqrt{3}$ c) 12 d) 6	
10	If $\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$, then the value of $ab - cd$ is:	1
	a) 16 b) -16 c) 4 d) -4	
11	The value of k for which the function $f(x) = \begin{cases} \frac{x^3 - 8}{x-2} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$ is continuous at $x = 2$ is _____.	1
	(a) 4 (b) -4 (c) 12 (d) 0	
12	The value of $\frac{dy}{dx}$ when $x = a \cos^3 \theta$, $y = a \sin^3 \theta$ is :	1
	a) $\tan \theta$ b) $-\tan \theta$ c) $\cot \theta$ d) $-\cot \theta$	
13	The corner points of the feasible region determined by a system of linear equations with $Z = ax + by$ where $a, b > 0$ are $(0,0)$, $(2,4)$, $(4,0)$ and $(0,5)$. The relation between a and b so that the maximum of Z occurs at both $(2,4)$ and $(4,0)$ is:	1
	a) $a = 2b$ b) $2a = b$ c) $a = b$ d) $3a = b$	
14	The derivative of x^x is	1
	a) xx^{x-1} b) $x^x(1 + \log x)$ c) $x^x \log x$ d) $x^x - \log x$	
15	If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B/A) = 0.6$, then $P(A \cup B) =$ ----	1
	a) 0.96 b) 1.44 c) 1.04 d) 0.24	
16	Find the degree of the differential equation: $4 \frac{\left(\frac{d^2y}{dx^2}\right)^3}{\frac{d^3y}{dx^3}} + \frac{d^3y}{dx^3} = x^2 - 1$	1
	a) 2 b) 3 c) 1 d) Not defined	
17	Sam plotted three points A (2, -3), B (x , -1) and C (0, 4) on a graph sheet. The value of x that makes the points collinear is _____.	1
	a) -10 b) 10 c) $\frac{10}{7}$ d) $-\frac{10}{7}$	

18 Identify the function from the graph.

a) $\sec^{-1} x$ b) $\cosec^{-1} x$

c) $\tan^{-1} x$

d) $\cot^{-1} x$



1

19 Ans: a

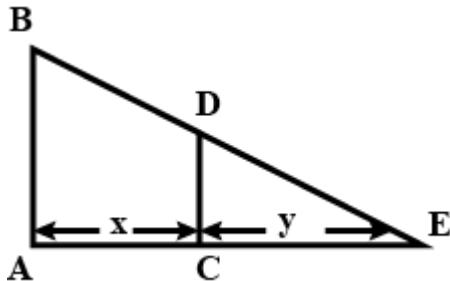
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20 Ans: b

1

SECTION B

21



Let AB be the lamp post. Let at any time t, the man CD be at a distance x metres from the lamp post and y metres the length of the shadow.

Given:

$$\frac{dx}{dt} = 5 \text{ km/hr}$$

Clearly $\triangle ABC$ and $\triangle CDE$ are similar

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{6}{2} = \frac{x+y}{y}$$

$$3y = x + y$$

$$2y = x$$

$$2 \frac{dy}{dt} = \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{5}{2}, \text{ m/min}$$

1

½

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22	$ a = b = c = 1, \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$ $ \vec{a} + \vec{b} + \vec{c} ^2 = a ^2 + b ^2 + c ^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$ $= 3$ $ \vec{a} + \vec{b} + \vec{c} = \sqrt{3}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
23	$y = \cos^{-1}(x^2 - 4) \Rightarrow \cos y = x^2 - 4$ <i>i.e.</i> $-1 \leq x^2 - 4 \leq 1$ (since $-1 \leq \cos y \leq 1$) $\Rightarrow 3 \leq x^2 \leq 5$ $\Rightarrow \sqrt{3} \leq x \leq \sqrt{5}$ $\Rightarrow x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$ OR $\sin^{-1} \left[\cos(-\frac{17\pi}{8}) \right] = \sin^{-1} \left[\cos(\frac{17\pi}{8}) \right]$ $= \sin^{-1} \left[\cos(\frac{\pi}{8}) \right]$ $= \sin^{-1} \left[\sin(\frac{3\pi}{8}) \right]$ $= \frac{3\pi}{8}$	$\frac{1}{2}$ $\frac{1}{2}$
24	$y = \sin(\log x)$ $y_1 = \cos(\log x) \frac{d}{dx} (\log x)$ $y_1 = \cos(\log x) \cdot \frac{1}{x}$ $\therefore xy_1 = \cos(\log x)$ <p>Differentiating both sides with respect to x, we get</p> $xy_2 + y_1(1) = -\sin(\log x) \cdot \frac{1}{x}$ $\Rightarrow x[xy_2 + y_1] = -\sin(\log x)$ $\Rightarrow x^2y_2 + xy_1 = -y$ $\Rightarrow x^2y_2 + xy_1 + y = 0$	$\frac{1}{2}$ $\frac{1}{2}$

25

Given cartesian form of line as:

$$\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = \mu$$

\therefore General point on line is $(3\mu - 2, 2\mu - 1, 2\mu + 3)$

Since distance of points on line from $P(1, 3, 3)$ is 5 units.

$$\therefore \sqrt{(3\mu - 2 - 1)^2 + (2\mu - 1 - 3)^2 + (2\mu + 3 - 3)^2} = 5$$

$$\Rightarrow (3\mu - 3)^2 + (2\mu - 4)^2 + (2\mu)^2 = 25$$

$$\Rightarrow 17\mu^2 - 34\mu = 0 \Rightarrow 17\mu(\mu - 2) = 0 \Rightarrow \mu = 0, 2$$

Required point on line is $(-2, -1, 3)$ for $\mu = 0$, or $(4, 3, 7)$ for $\mu = 2$

OR

$$(i) \vec{a} = \vec{i} - \vec{j} + 7\vec{k}$$

$$\vec{b} = 5\vec{i} - \vec{j} + \lambda\vec{k}$$

$$(\vec{a} + \vec{b}) = \hat{i} - \hat{j} + 7\hat{k} + 5\hat{i} - \hat{j} + \lambda\hat{k}$$

$$\Rightarrow \vec{a} + \vec{b} = 6\hat{i} - 2\hat{j} + (7 + \lambda)\hat{k}$$

$$\vec{a} - \vec{b} = \hat{i} - \hat{j} + 7\hat{k} - (5\hat{i} - \hat{j} + \lambda\hat{k})$$

$$\Rightarrow \vec{a} - \vec{b} = -4\hat{i} + 0\hat{j} + (7 - \lambda)\hat{k}$$

$$\text{Now } (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})$$

$$= (6\hat{i} - 2\hat{j} + (7 + \lambda)\hat{k}) \cdot (-4\hat{i} + 0\hat{j} + (7 - \lambda)\hat{k})$$

Since these two vectors are orthogonal, their dot product is zero.

$$\Rightarrow (6 \times -4) + (-2 \times 0) + ((7 + \lambda) \times (7 - \lambda)) = 0$$

$$\Rightarrow -24 + 0 + (49 - \lambda^2) = 0$$

$$\Rightarrow \lambda^2 = 25$$

$$\Rightarrow \lambda = \pm 5$$

1

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SECTION C

26 We have, $A_1 : A_2 : A_3 = 4 : 4 : 2$

$$P(A_1) = \frac{4}{10}, P(A_2) = \frac{4}{10} \text{ and } P(A_3) = \frac{2}{10}$$

$\frac{1}{2}$

where A_1, A_2 and A_3 denote the three types of flower seed.

Let E be the event that a seed germinates and \bar{E} be the event that a seed does not germinate.

$$\therefore P(E/A_1) = \frac{45}{100}, P(E/A_2) = \frac{60}{100}, P(E/A_3) = \frac{35}{100}$$

$$\text{And } \therefore P(\bar{E}/A_1) = \frac{55}{100}, P(\bar{E}/A_2) = \frac{40}{100}, P(\bar{E}/A_3) = \frac{65}{100}$$

1

$$P(A_2/\bar{E}) = \frac{P(A_2) \cdot P(\bar{E}/A_2)}{P(A_1) \cdot P(\bar{E}/A_1) + P(A_2) \cdot P(\bar{E}/A_2) + P(A_3) \cdot P(\bar{E}/A_3)}$$

$$\frac{\frac{4}{10} \cdot \frac{40}{100}}{\frac{4}{10} \cdot \frac{55}{100} + \frac{4}{10} \cdot \frac{40}{100} + \frac{2}{10} \cdot \frac{65}{100}} = \frac{\frac{160}{1000}}{\frac{220}{1000} + \frac{160}{1000} + \frac{130}{1000}}$$

$$= \frac{160/1000}{510/1000} = \frac{16}{51}$$

1

OR

Let X denote the number of doublets. Possible doublets are

$(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)$

Clearly, X can take the values 0, 1, 2 or 3.

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Probability of getting a doublet = $6/36 - 1/6$

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Probability of not getting a doublet = $1 - 1/6 - 5/6$

Now,

½

$$P(X = 0) = P(\text{no doublet}) = 5/6 \times 5/6 \times 5/6 = 125/216$$

½

$P(X = 1) = P(\text{one doublet and two non-doublets})$

$$= \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = 3 \left(\frac{1}{6} \times \frac{5^2}{6^2} \right) = \frac{75}{216}$$

½

$P(X = 2) = P(\text{two doublets and one non-doublet})$

$$= \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} + \frac{1}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{1}{6} \times \frac{1}{6} = 3 \left(\frac{1}{6^2} \times \frac{5}{6} \right) = \frac{15}{216}$$

½

$$\text{and } P(X = 3) = P(\text{three doublets}) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}.$$

½

Thus, the required probability distribution is

X	0	1	2	3
$P(X)$	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$

½

27

$$\text{Consider, } I = \int \sqrt{\tan x} + \sqrt{\cot x} dx$$

$$= \int \sqrt{\frac{\sin x}{\cos x}} + \sqrt{\frac{\cos x}{\sin x}} dx$$

$$= \int \frac{\sin x + \cos x}{\sqrt{\cos x \sin x}} dx$$

$$\text{Let } \sin x - \cos x = t$$

$$\Rightarrow (\cos x + \sin x) dx = dt$$

$$\text{Also, } (\sin x - \cos x)^2 = t^2$$

$$\Rightarrow 1 - 2 \sin x \cos x = t^2$$

$$\Rightarrow \sin x \cos x = \frac{1-t^2}{2}$$

$$\Rightarrow \sqrt{\sin x \cos x} = \frac{\sqrt{1-t^2}}{\sqrt{2}}$$

$$\therefore \int \frac{\sin x + \cos x}{\sqrt{\cos x \sin x}} dx = \sqrt{2} \int \frac{1}{\sqrt{1-t^2}} dt$$

$$= \sqrt{2} \sin^{-1} t$$

$$= \sqrt{2} \sin^{-1}(\sin x - \cos x)$$

½

1

½

1

28

$$F(\lambda x, \lambda y) = \frac{2\lambda y e^{\lambda x/\lambda y}}{2\lambda x e^{\lambda x/\lambda y} - \lambda y} = \frac{2y e^{x/y}}{2x e^{x/y} - y} = F(x, y)$$

Hence, it is a homogenous function.

$$\text{Putting } \frac{x}{y} = v$$

$$\frac{dx}{dy} = v + y \frac{dv}{dy}$$

$$\frac{dx}{dy} = \frac{2v e^v - 1}{2e^v}$$

$$v + y \frac{dv}{dy} = \frac{2v e^v - 1}{2e^v}$$

$$\frac{dy}{y} = -2e^v dv$$

$$\log y = -2e^v + C$$

$$\log y + 2e^{x/y} = C$$

1

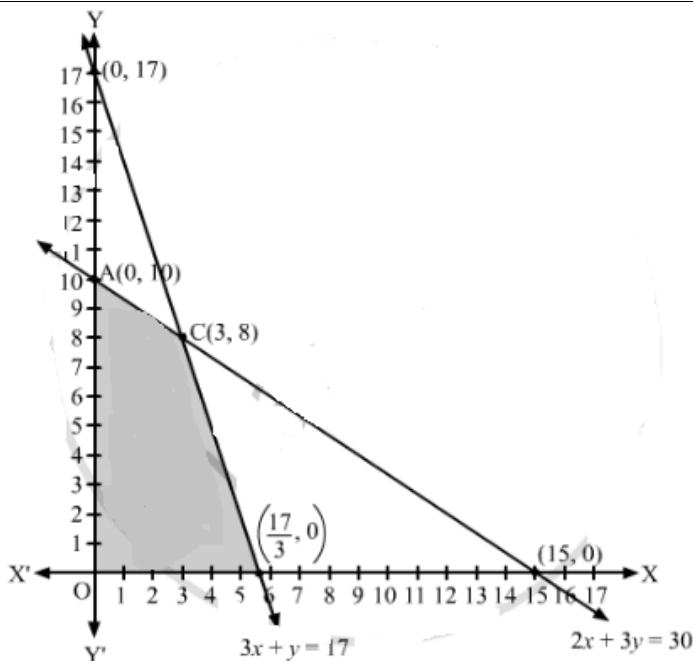
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1

OR

30

2



Corner Point $Z = 100x + 120y$

1

$$(0, 0) \quad 100 \times 0 + 120 \times 0 = 0$$

$$(0, 10) \quad 100 \times 0 + 120 \times 10 = 1200$$

$$\left(\frac{17}{3}, 0\right) \quad 100 \times \frac{17}{3} + 120 \times 0 = \frac{1700}{3}$$

$$(3, 8) \quad 100 \times 3 + 120 \times 8 = 1260 \rightarrow \text{Maximum}$$

The maximum value of Z is 1260 at $x = 3, y = 8$.

31

$$\text{Given, } I = \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx \quad \dots \dots \dots (1)$$

$$I = \int_0^{\pi} \frac{(\pi - x) \sin(\pi - x)}{1 + \cos^2(\pi - x)} dx = \int_0^{\pi} \frac{(\pi - x) \sin x}{1 + \cos^2 x} dx \quad \dots \dots \dots (2)$$

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$$(1) + (2)$$

$$\Rightarrow 2I = \int_0^{\pi} \frac{\pi \sin x}{1 + \cos^2 x} dx = \pi \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx$$

$$\Rightarrow I = \frac{\pi}{2} \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx$$

1

We have, $\Rightarrow I = \frac{\pi}{2} \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx$,

Put $\cos x = t \Rightarrow -\sin x dx = dt$,

When $x = 0 \Rightarrow t = 1, x = \pi \Rightarrow t = -1$

$$\begin{aligned} I &= \frac{\pi}{2} \int_1^{-1} \frac{-dt}{1+t^2} = -\frac{\pi}{2} (\tan^{-1} t) \Big|_1^{-1} \\ &= -\frac{\pi}{2} (\tan^{-1}(-1) - \tan^{-1}(1)) \\ &= -\frac{\pi}{2} \left(-\frac{\pi}{4} - \frac{\pi}{4} \right) = \frac{\pi^2}{4} \end{aligned}$$

1

$\frac{1}{2}$

OR

$$I = \int_1^3 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{4-x}} dx \quad \dots \quad 1$$

$$I = \int_1^3 \frac{\sqrt{4-x}}{\sqrt{4-x} + \sqrt{x}} dx \quad \dots \quad 2 \quad \left(\int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right)$$

1

$$1 + 2 \Rightarrow 2I = \int_1^3 \frac{\sqrt{x} + \sqrt{4-x}}{\sqrt{4-x} + \sqrt{x}} dx$$

1

$$I = \frac{1}{2} \int_1^3 1 dx$$

1

$$I = 1$$

SECTION D

32

Let the cartesian equation of line passing through $(1, 2, -4)$ be

$$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c} \dots (i)$$

 $\frac{1}{2}$

Given lines are

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \dots (ii)$$

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5} \dots (iii)$$

Obviously parallel vectors \vec{b}_1, \vec{b}_2 , and \vec{b}_3 of (i), (ii) and (iii) respectively are given as

$$\vec{b}_1 = a\hat{i} + b\hat{j} + c\hat{k}; \vec{b}_2 = 3\hat{i} - 16\hat{j} + 7\hat{k}; \vec{b}_3 = 3\hat{i} + 8\hat{j} - 5\hat{k}$$

According to question

$$(i) \perp (ii) \Rightarrow \vec{b}_1 \perp \vec{b}_2 \Rightarrow \vec{b}_1 \cdot \vec{b}_2 = 0$$

$$(i) \perp (iii) \Rightarrow \vec{b}_1 \perp \vec{b}_3 \Rightarrow \vec{b}_1 \cdot \vec{b}_3 = 0$$

$$\text{Hence, } 3a - 16b + 7c = 0 \dots (iv)$$

$$\text{and } 3a + 8b - 5c = 0 \dots (v)$$

1

1

From equation (iv) and (v), we get

$$\frac{a}{80-56} = \frac{b}{21+15} = \frac{c}{24+48}$$

$$\Rightarrow \frac{a}{24} = \frac{b}{36} = \frac{c}{72} \Rightarrow \frac{a}{2} = \frac{b}{3} = \frac{c}{6} = \lambda \text{ (say)}$$

$$\Rightarrow a = 2\lambda, b = 3\lambda, c = 6\lambda$$

1

Putting the value of a, b, c in (i), we get the required cartesian equation of line as

$$\frac{x-1}{2\lambda} = \frac{y-2}{3\lambda} = \frac{z+4}{6\lambda} \Rightarrow \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

1

Equation of the line passing through the point $(0, -2, 4)$ and parallel to the above line is $\frac{x}{2} = \frac{y+2}{3} = \frac{z-4}{6}$

 $\frac{1}{2}$

OR

Let M be the foot of the perpendicular drawn from the point A (1, 2, 1) to the line joining P (1, 4, 6) and Q (5, 4, 4).

Equation of a line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Equation of the required line passing through P (1, 4, 6) and Q(5, 4, 4) is

$$\frac{x-1}{4} = \frac{y-4}{0} = \frac{z-6}{-2} = \lambda$$

$$x = 4\lambda + 1; y = 4; z = -2\lambda + 6$$

Coordinates of M are $(4\lambda + 1, 4, -2\lambda + 6)$ (1)

The direction ratios of AM are

$$4\lambda + 1 - 1, 4 - 2, -2\lambda + 6 - 1$$

i.e. $4\lambda, 2, -2\lambda + 5$

The direction ratios of given line are 4,0,-2

Since AM is perpendicular to the given line

$$\therefore 4(4\lambda) + 0(2) + (-2)(-2\lambda + 5) = 0$$

$$\therefore \lambda = \frac{1}{2}$$

Putting $\lambda = \frac{1}{2}$ in (i), the coordinates of M are (3,4,5)

Coordinates of Image (5, 6, 9)

33

Given that

$$x+y+z=10$$

$$2x+y=13$$

$$x+y=4z$$

Rewrite the above equations as,

$$x+y+z=10$$

$$2x+y=13$$

$$x+y - 4z=0$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 13 \\ 0 \end{bmatrix}$$

 $\Rightarrow AX = B$, where,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & -4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 10 \\ 13 \\ 0 \end{bmatrix}$$

Thus, $X = A^{-1}B$

Let us find the determinant of A:

$$|A| = 1(-4 - 0) - 1(-8 - 0) + 1(2 - 1) = -4 + 8 + 1 = 5$$

$$\text{Adj } A = \begin{bmatrix} -4 & 5 & 1 \\ 8 & -5 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{1}{5} \begin{bmatrix} -4 & 5 & 1 \\ 8 & -5 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

Thus $X = A^{-1}B$

$$\Rightarrow X = \frac{1}{5} \begin{bmatrix} -4 & 5 & 1 \\ 8 & -5 & 2 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 13 \\ 0 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 25 \\ 15 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$$

$$x = 5, y = 3, z = 2$$

1

1

1

1

1

OR

$$B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 4 & -1 & 5 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 + 4 - 0 & 2 - 2 + 0 & -4 + 4 + 0 \\ -4 - 12 + 16 & 4 + 6 - 4 & -8 - 12 + 20 \\ 0 - 4 + 8 & 0 - 2 + 2 & 0 - 4 + 10 \end{bmatrix}$$

$$BA = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

2

Now, we can see that it is $BA = 6I$. where I is the unit Matrix

$$\text{Or, } B^{-1} = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 4 & -1 & 5 \end{bmatrix}$$

Now, $BX = C$

$$\text{where, } B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } C = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

1

$$\therefore X = B^{-1}C$$

$$\Rightarrow X = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 4 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

1

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 6 + 34 - 28 \\ -12 + 34 - 28 \\ 6 - 17 + 35 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$$

$$\therefore x = 2, y = -1 \text{ and } z = 4.$$

1

34

Let $(a, b) \in N \times N$
then,

$$\begin{aligned} \therefore a^2 + b^2 &= a^2 + b^2 \\ \therefore (a, b) R (a, b) \end{aligned}$$

Hence R is reflexive.

Let $(a, b), (c, d) \in N \times N$ be such that

$$\begin{aligned} (a, b) R (c, d) \\ \Rightarrow a^2 + d^2 = b^2 + c^2 \\ \Rightarrow c^2 + b^2 = d^2 + a^2 \\ \Rightarrow (c, d) R (a, b) \end{aligned}$$

Hence, R is symmetric.

Let $(a, b), (c, d), (e, f) \in N \times N$ be such that

$(a, b) R (c, d), (c, d) R (e, f)$.

$$\Rightarrow a^2 + d^2 = b^2 + c^2 \dots(i)$$

$$\text{and } c^2 + f^2 = d^2 + e^2 \dots(ii)$$

Adding eqn. (i) and (ii),

$$\Rightarrow a^2 + d^2 + c^2 + f^2 = b^2 + c^2 + d^2 + e^2$$

$$\Rightarrow a^2 + f^2 = b^2 + e^2$$

$$\Rightarrow (a, b) R (e, f)$$

Hence, R is transitive

$\therefore R$ is an equivalence relation.

1

1
½

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1

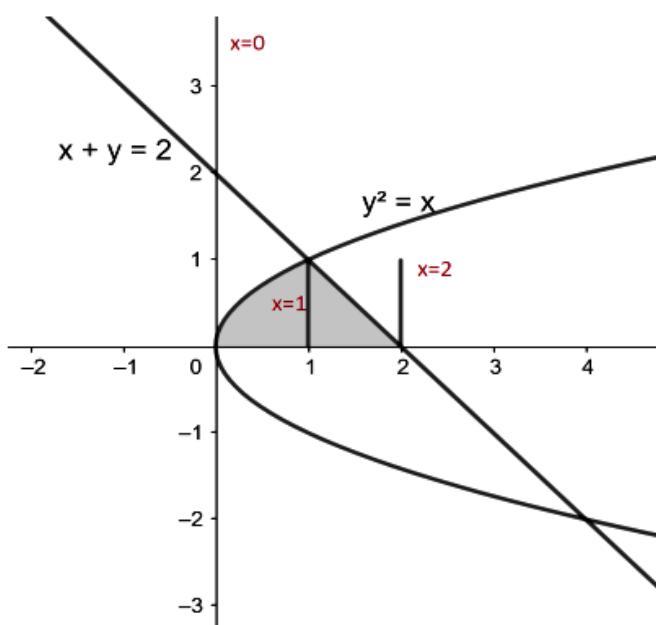
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35

Solving $x + y = 2$ and $y^2 = x$ simultaneously, we get the points of intersection as $(1, 1)$, $(4, -2)$.



The required area = the shaded area = $\int_0^1 \sqrt{x} dx + \int_1^2 (2-x) dx$

	$= \frac{2}{3} [x^{\frac{3}{2}}]_0^1 + [2x - \frac{x^2}{2}]_1^2$ $= \frac{2}{3} + \frac{1}{2} = \frac{7}{6}$ square units	1
	SECTION E	
36	$f(x) = -16x^2 + mx + 3, 0 \leq x \leq 10.$ $f'(x) = -32x + m.$ 4 is a critical point. Hence $f'(4) = 0 \Rightarrow m = 32 \times 4$ $m = 128$ b) $(0, 4)$ is the interval in which the function is strictly increasing and in $(4, 10)$ it is strictly decreasing.	2
37	a) $C = 1200t + \frac{3v^2}{16}t \therefore C = 1200\left(\frac{S}{v}\right) + \frac{3v^2}{16}\left(\frac{S}{v}\right)$ $\therefore C = 1200\left(\frac{S}{v}\right) + \frac{3vS}{16}$ b) $\frac{dC}{dv} = 0 \Rightarrow -1200 \frac{S}{v^2} + \frac{3S}{16} = 0 \therefore v = 80$ c) Use first derivative test, $\therefore v = \frac{80 \text{ km}}{\text{hr}}$ OR $\frac{d^2C}{dv^2} = 2400 \times \frac{S}{v^3} \therefore \frac{d^2C}{dv^2} = 2400 \times \frac{S}{v^3} > 0$ Minimum cost if distance is 100km = Rs 3000	1
38	a. $\frac{3}{20}$ b. $\frac{13}{30}$ c. $\frac{1}{6}$ OR $\frac{5}{6}$	1 1 2